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How to Use This Addenda

Make sure you're ready to teach by noting the **Necessary Materials and Pre-Lesson Prep** you will need to gather or complete prior to the lesson

Find high-leverage instructional moves in the **Lesson Look Fors**. This is what leaders should see when observing your instruction

Note how your lesson objective ties to your state **Standards**

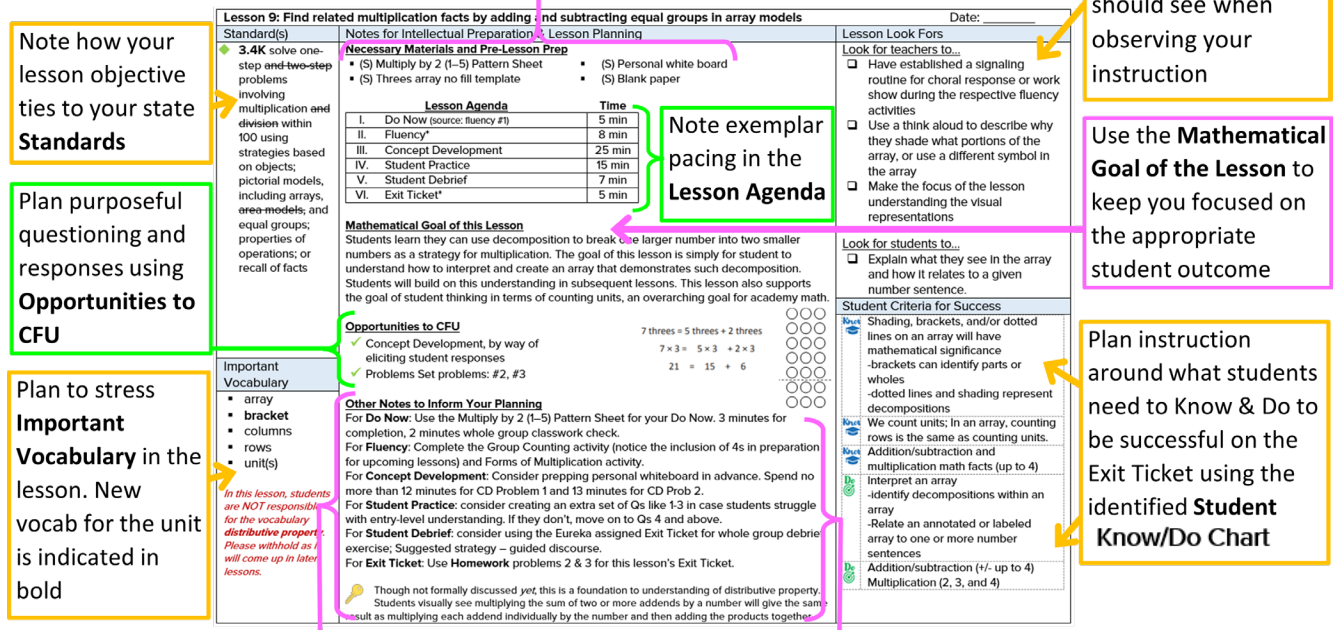
Note exemplar pacing in the **Lesson Agenda**

Use the **Mathematical Goal of the Lesson** to keep you focused on the appropriate student outcome

Plan purposeful questioning and responses using **Opportunities to CFU**

Plan to stress **Important Vocabulary** in the lesson. New vocab for the unit is indicated in bold

Plan instruction around what students need to Know & Do to be successful on the Exit Ticket using the identified **Student Know/Do Chart**



Find recommended lesson modifications, content knowledge boosters, and/or high-leverage instructional moves that may not be in your Teacher Edition located in **Other Notes to Inform Your Planning**

UNIT SYNOPSIS

This unit expands on students' understanding of solving systems of linear equations and inequalities from Algebra 1 where they focused on two variable linear systems and systems of one linear equation and one quadratic equation. In this unit, students will extend their abilities to include three variables, and beyond, systems of linear equations and inequalities. Students will use solving by graphing, substitution, and elimination as well as learning how to solve systems using matrices on calculators or computers. In addition to solving systems of linear inequalities, students will be introduced to optimization using systems of inequalities. Systems of inequalities are often used to model real world situations and the optimization lesson introduces using feasible regions to find the maximum and minimum outputs of a function that is governed by given constraints.

Solving Equations and Inequalities: Solving an equation is the process of rewriting the equation to make what it says about its variable(s) as simple as possible. Properties of numbers and equality can be used to transform an equation (or inequality) into equivalent, simpler equations (or inequalities) in order to find solutions. Useful information about equations and inequalities (including solutions) can be found by analyzing graphs or tables. The numbers and types of solutions vary predictably, based on the type of equation.

- To solve a system of equations, find a set of values that replace the variables in the equations and make each equation true.
- You can solve a system of equations by writing equivalent systems until the value of one variable is clear. Then substitute to find the value(s) of the other variable.
- You can solve a system of inequalities in more than one way. Graphing the solution is usually the most appropriate method. The solution is the set of all points that are solutions of each inequality in the system.
- To solve systems of three equations in three variables, you can use some of the same algebraic methods you used to solve systems of two equations in two variables.
- You can use a matrix to represent and solve a system of equations.

Functions: A function is a relationship between variables in which each value of the input variable is associated with a unique value of the output variable. Functions can be represented in a variety of ways, such as graphs, tables, equations, or words. Each representation is particularly useful in certain situations. Some important families of functions are developed through transformations of the simplest form of the function.

- Some real-world problems involve multiple linear relationships. Optimizing accounts for all of these linear relationships and gives the solution to the problem.

Equivalence: A single quantity may be represented by many different expressions. The facts about a quantity may be expressed by many different equations (or inequalities).

- You can solve a system of equations by writing equivalent systems until the value of one variable is clear. Then substitute to find the value(s) of the other variable.
- To solve systems of three equations in three variables, you can use some of the same algebraic methods you used to solve systems of two equations in two variables.

Misconceptions:

- Some students may think that the x- and y-values in the solution to a system of two linear equations in two variables can never be equal in value.
- Some students may think the x-, y-, and z-values in the solution to a system of three linear equations in three variables can never be equal in value.
- Some students may think that a system of three equations in three variables can be represented on a coordinate plane rather than on a three-dimensional coordinate system.

Underdeveloped Concepts:
- Some students may not understand the differences between linear equations in one variable and linear equations in two variables.

Key Questions:
- How does knowing more than one solution strategy build mathematical flexibility?
- How can systems of equations be used to represent relationships between quantities?
- What methods can be used to write systems of equations?
- What methods can be used to solve systems of equations?
- How is solving a system of linear inequalities different from solving a system of linear equations?

CONTENT STANDARDS

Below are the standards addressed in this unit.

Readiness Standards	Supporting Standards
2A.3A Formulate systems of equations, including systems consisting of three linear equations in three variables and systems consisting of two equations~~, the first linear and the second quadratic.~~	**2A.3E** Formulate systems of at least two linear inequalities in two variables.
2A.3B Solve systems of three linear equations in three variables by using technology with matrices and substitution. (**NOTE:** AP readiness does not support inclusion of Gaussian elimination)	**2A.3F** Solve systems of two or more linear inequalities in two variables.
	2A.3G Determine possible solutions in the solution set of systems of two or more linear inequalities in two variables.

Focus on Disciplinary Literacy	Mathematical Process Standard (F) – Analyze mathematical relationships to connect and communicate mathematical ideas.
	Mathematical Process Standard (G) – Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

LEARNING SUPPORTS BY LESSON

There is a checkmark for the math support if the lesson	Lessons → Math Supports	L1	L2	L3	L4	L5	L6	L7	L8	L9
makes a connection to prior content or from a previous unit or academic year	Access Prior Knowledge	✓	✓	✓	✓	✓	✓	✓		✓
uses familiar contexts or experiences to make the learning relevant to students	Real-World Connections		✓	✓	✓	✓	✓			✓
makes use of graphic organizers	Graphic Organizers		✓				✓		✓	✓
includes tools like rulers, protractors, patty paper, algebra tiles, etc.	Tools or Manipulatives							✓	✓	✓
incorporates tables, reference charts, displays, pictures, models, or color-coding	Visual Aids	✓	✓			✓	✓	✓	✓	
includes definitions, examples vs. nonexamples, cognates, etc.	Vocabulary Supports	✓	✓	✓	✓	✓	✓	✓	✓	✓
includes strategies that support language development										
asks students to discuss with their partner to prepare for whole class discussion	- Turn and Talk	✓	✓	✓	✓	✓	✓			✓
teacher facilitates a whole class discussion to debrief key learnings	- Guided Discussion	✓	✓	✓	✓	✓	✓	✓	✓	✓
asks students to think independently, test their idea with a partner, and share whole group	- Think, Pair, Share	✓								
includes sentence stems to support students with explanations	- Sentence Stems									
provides opportunities for students to work with a partner or a group	Peer Collaboration	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses mnemonics such as SohCahToa	Mnemonics									
includes websites or equipment that enhances the lesson	Technological Support	✓	✓	✓		✓	✓	✓	✓	✓
content can be presented in different forms										
uses hands-on tools or manipulatives to represent the math	- Concrete									
uses drawings to represent the math	- Pictorial	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses numbers and number sentences to represent the math	- Abstract	✓	✓	✓	✓	✓	✓	✓	✓	✓

The EFFL Model

Before You EFFL!

Here are helpful resources that you guide you in the right direction before your EFFL lesson!

Why Should We EFFL?

The article advocates for the Experience First, Formalize Later (EFFL) teaching model, emphasizing its effectiveness in fostering deep understanding and flexible thinking in students. The author compares traditional teaching to a game of "Simon Says," where students merely mimic instructions without grasping underlying concepts. In contrast, EFFL encourages students to engage actively with problems, enhancing their ability to understand and apply math concepts creatively.

Tips for Lesson Planning

The article offers practical advice for effective lesson planning beyond the exhaustive and overly detailed approaches often emphasized during teacher training. It underscores the importance of thoughtful preparation but rejects the notion that teachers need to script every minute or detail of a class session.

Making the Most of Your EFFL Lesson Debrief

The article discusses the significance of the debriefing phase in the Experience First, Formalize Later (EFFL) lesson model, emphasizing its role in reinforcing learning and highlighting student contributions. The debrief session is seen as crucial for integrating academic vocabulary, emphasizing key lesson understandings, and valuing students' mathematical insights.

While You EFFL!

While each lesson may be unique in context and skills, all lessons benefit from the following practices:

Teacher Look Fors:

- ❑ Utilizing the Do Now to spark students' interest in the Activity.
- ❑ Use questioning to promote small group discussion and exploration, guided by monitoring questions.
- ❑ Connects Experience First to formal concepts using a colored pencil/pen to take notes along the margin during the Debrief.
- ❑ Facilitates whole-class discussions for students to reflect, share insights, and provides feedback that reinforces key concepts.
- ❑ Tracks time to adapt lesson pacing and support based on student response and engagement.

Students Look Fors:

- ❑ In the Activity, students engage in group work and discourse.
- ❑ Exploring the activity, testing hypotheses and approaches (trial & error).
- ❑ Take notes on key ideas and concepts using different colored pencil/pen to take notes along the margin.
- ❑ Share thoughts and ideas that demonstrate their approach to their work.

Other considerations

- During the **Experience First** phase, if most of your students seem stuck or disengaged, take a moment to pause, reset, and provide clear instructions. Some problems of the Activity are more suitable to do a whole-class discussion as a means to save some instructional time for Student Practice or the Exit Ticket. You are encouraged to adapt the EFFL (Experience First, Formalize Later) process to meet your students' needs while maintaining a focus on student-centered instruction.

ROADMAP

AT A GLANCE: Unit 3 – Systems of Linear Equations and Inequalities			
Day	Date	Lesson	Lesson Title
1		1	Linear Systems
2		2	Number of Solutions
3		3	Elimination
4			*Unit 3 Success Day 1 – Review topics from 3.1 - 3.3*
5		4	Larger Systems of Equations
6		5	Systems of Inequalities
7		6	Optimization Using Systems of Inequalities
8			*Unit 3 Success Day 2 – Review topics from 3.4 - 3.6*
9		7	Introduction to Matrices
10		8	Solving Linear Systems with Matrix Inverses & RREF
11		9	Writing and Solving Linear Systems with Matrix Inverses
12			*Unit 3 Success Day 3 – Use as needed based on your data*
13			*Unit 3 Success Day 4 – Unit Assessment Review*
14			End of Unit 3 Assessment

Date: _____

Lesson 1: Linear Systems

Standard(s)

- **2A.3A** Formulate systems of equations, ~~including systems consisting of three linear equations in three variables and systems consisting of two equations, the first linear and the second quadratic.~~
- **2A.3B** Solve systems of ~~three~~ linear equations ~~in three variables~~ by using ~~technology with matrices and~~ substitution.

Important Vocabulary

- system of linear equations
- ordered pair
- **empty set, ∅**
- **standard form for systems of equations**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- Red and Blue "sticker dots" or equivalent per student

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Debrief (10 min)
- Student Practice (15 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to find solutions to two-variable linear systems by graphing or by using substitution.

Opportunities to CFU

- ✓ INM #5-6
- ✓ Debrief
- ✓ Student Practice #3-6

Focus on Disciplinary Literacy

INM + Student Practice #1 & 2

Other Notes to Inform Your Planning

Your students should have seen linear systems in Algebra 1, but we're going to begin by investigating what it means to be a solution to an equation and then also a system. In this lesson we will only be solving systems that have one solution. In the next lesson we will investigate systems with infinite solutions or no solutions. We will focus on solving by graphing and substitution today, elimination comes later in the unit.
This lesson calls for a large poster graph, red sticker dots, and blue sticker dots. Each student will need one red and one blue sticker dot. You can put some at each group before class starts to save some time. (You could do this activity with two colored markers on your whiteboard instead.)

Lesson Look Fors

Look for teachers to…

- ❑ Have each student participate in plotting points in the INM.
- ❑ Build on students' prior knowledge of solving with graphs.

Look for students to…

- ❑ Graph a system of lines and interpret the solution of the system in context.
- ❑ Use substitution to solve systems algebraically.

Student Know/Do Chart

Students can

Solve a system of linear equations by graphing and finding the point of intersection.

Solve a system of linear equations algebraically using substitution to find the solution for one variable and then plugging back into an equation to solve for the second variable.

Know that for an ordered pair to be the solution of a system of equations, the ordered pair needs to be the point of intersection of the graphs or make true statements when plugged into both equations.

Date: _____

Lesson 2: Number of Solutions

Standard(s)

- **2A.3A** Formulate systems of equations, ~~including systems consisting of three linear equations in three variables and systems consisting of two equations, the first linear and the second quadratic.~~
- **2A.3B** Solve systems of ~~three~~ linear equations ~~in three variables~~ by using ~~technology with matrices and~~ substitution.

Important Vocabulary

- system of linear equations
- empty set, ∅
- **consistent dependent, consistent independent**
- **inconsistent**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook

Lesson Structure:

- Do Now (5 min)
- INM (20 min)
- Debrief (10 min)
- Student Practice (20 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to determine if a system has one solution, no solution, or infinitely many solutions. Students will also identify the features that make a system consistent or inconsistent.

Opportunities to CFU

✓ After INM #2 & 3

✓ Student Practice #1-5

Focus on Disciplinary Literacy

INM + Student Practice #2 & 3

Other Notes to Inform Your Planning

Today students will find that linear systems can have one solution, no solutions, or infinite solutions. This activity is set up so that students can use what they learned in the previous lesson to solve the first scenario. There is only one solution in the system described in question 1. Encourage students to think about what we talked about in the last lesson. Focus their attention on the graph they've made. When we transition into questions 2 and 3, we want to continue looking at the graphs and thinking about what they're telling us.

Lesson Look Fors

Look for teachers to…

- ❑ Use the guiding questions to keep the students thinking and avoid answer giving.
- ❑ Encourage students to draw upon what they know about what needs to be true for lines to intersect or never intersect.

Look for students to…

- ❑ Apply what they know about modeling situations with linear equations to compare two companies' costs.
- ❑ Consider slope or rate in determining if lines will intersect, not just what can be seen on a graph.

Student Know/Do Chart

Students can

Do Determine the number of solutions a linear system has.

Know Know that consistent systems have lines that intersect. Consistent independent systems intersect and share one point called a solution. Consistent dependent systems are the same line and have infinitely many solutions.

Know Know that inconsistent systems have no solution and share no common points. Systems of equations of parallel lines are inconsistent.

Date: _____
Lesson 3: Elimination

Standard(s)

- **2A.3A** Formulate systems of equations, ~~including systems consisting of three linear equations in three variables and systems consisting of two equations, the first linear and the second quadratic.~~
- **2A.3B** Solve systems of ~~three~~ linear equations ~~in three variables~~ by using ~~technology with matrices and~~ substitution.

Important Vocabulary

- system of linear equations
- empty set, ∅
- **elimination method**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook

Lesson Structure:

- Do Now (5 min)
- INM (20 min)
- Debrief (10 min)
- Student Practice (20 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to solve linear systems using elimination. Students have already been solving algebraically with substitution, which is a step they will apply in elimination. They may have also solved systems with substitution and elimination in Algebra 1. Students will then use elimination strategies to determine if a system has 0, 1, or infinite solutions.

Opportunities to CFU

- ✓ INM problems #1-2
- ✓ Debrief
- ✓ Student Practice #1-3

Focus on Disciplinary Literacy

Student Practice #3, 5, & 6

Other Notes to Inform Your Planning

The goal of this lesson is that students use their problem-solving skills to solve linear systems without a step-by-step procedure. Instead, we want them thinking about how they would approach each scenario to best solve it by considering what they are given. It is our hope in this lesson that students won't jump to graphing in their calculators or to solving with substitution. While they could do that, this system is much faster to solve without those methods, which is one of the reasons we like elimination. Sometimes it's a lot simpler than other methods. We want students to discover/rediscover when elimination is the best method to use.

Lesson Look Fors

Look for teachers to…

- ❑ Use the guiding questions to keep the students thinking and avoid answer giving.
- ❑ Support students as they determine which variable to eliminate first. Question students on why that variable?

Look for students to…

- ❑ Write or convert equations so they are in the same form before attempting elimination.
- ❑ Select scalar multiples, sometimes negative ones, to create a multiple of an equation to use elimination.

Student Know/Do Chart

Students can

- Do: Solve a system of equations using elimination by first comparing the coefficient of the variables in each equation when they are in the standard form.
- Do: When needed, use scalar multiples to rewrite equivalent equations to use in elimination.
- Know: Know that elimination can be used to determine the number of solutions a system has. If all the variables are eliminated in one step, then there are no solutions. If a contradiction is left (ex: 2=0) or infinite solutions if an identity is left (ex: 2=2).

Date: _____

Lesson 4: Larger Systems of Equations

Standard(s)

- **2A.3A** Formulate systems of equations, including systems consisting of three linear equations in three variables ~~and systems consisting of two equations, the first linear and the second quadratic.~~
- **2A.3B** Solve systems of three linear equations in three variables by using ~~technology with matrices and~~ substitution.

Important Vocabulary

- empty set, ∅
- **ordered triple (x,y,z)**
- system of linear equations

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Debrief (5min)
- Student Practice (20 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to solve systems of equations with more than two variables. This lesson connects students' prior understanding of solving systems in two variables algebraically, to expand to systems with more unknowns.

Opportunities to CFU

- ✓ INM #2
- ✓ Debrief
- ✓ Student Practice #1, 4, & 5

Focus on Disciplinary Literacy

INM #2b

Other Notes to Inform Your Planning

So far this unit, we have focused on systems with two variables. This is the first lesson where students will work with three or more variables. Our goal is to simply get students used to thinking flexibly about working with systems. If they see a variable that can be solved for immediately, solve it! If they notice a variable can be eliminated immediately, eliminate it! We want to help students understand more deeply the idea that equations can be manipulated or scaled so that they are easier to work with. With three or more variables, students cannot just graph and look for the intersection of the lines. **This lesson is preceded by a success day, be sure your students are solid on substitution and elimination with that day.**

Lesson Look Fors

Look for teachers to…

- ❑ Give students time to struggle and remind them of their work in prior lessons.
- ❑ Highlight multiple students' methods of solving the system in #2 to illustrate there can be more than one approach.

Look for students to…

- ❑ Formulate and solve a system of equations to fit the information in the table.
- ❑ Give their solutions in context or as ordered triples.

Student Know/Do Chart

Students can

Do: Solve a system of three linear equations in three variables algebraically using substitution or elimination.

Know: A system of equations needs as many, or more, equations than the number of variables to have a unique solution.

Know: Not all systems of three linear equations in three variables have a unique solution.

Date: _____

Lesson 5: Systems of Inequalities

Standard(s)

- **2A.3E** Formulate systems of at least two linear inequalities in two variables.
- **2A.3F** Solve systems of two or more linear inequalities in two variables.
- **2A.3G** Determine possible solutions in the solution set of systems of two or more linear inequalities in two variables.

Important Vocabulary

- **system of linear inequalities**
- **solution set/region**
- **boundary line**
- **constraint**
- empty set, ∅

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbooks
- Each Student needs 3 colored pens (not red) or highlighters

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Debrief (10 min)
- Student Practice (15 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to graph systems of linear inequalities. Students will find ordered pairs that are solutions for systems of linear inequalities that they solve by graphing.

Opportunities to CFU

- ✓ After INM #4 and 5
- ✓ Debrief

Focus on Disciplinary Literacy

INM #5 & Student Practice #2b

Other Notes to Inform Your Planning

You'll notice that all the inequalities in the INM are ≤ or ≥ statements. This is helpful in case students aren't familiar with using a dashed versus a solid line. We will make sure to address this in the QuickNotes. Students will be working through the whole activity before the debrief. For the most part, they won't have much trouble with this activity. As you walk around and monitoring the groups, make sure to be checking that their inequalities are correct. It can be really frustrating for a student to spend a lot of time graphing only to find out they graphed the wrong inequality.

Lesson Look Fors

Look for teachers to…

- ❑ Discuss solid and dotted inequality graphs during the Debrief.
- ❑ Give students the space to complete the activity with limited interruptions but do make sure groups are correctly graphing the inequalities.

Look for students to…

- ❑ Graph and shade systems of linear inequalities using different colored pens or highlighters and clearly identify any shared shaded region.

Student Know/Do Chart

Students can

Do — Graph a system of linear inequalities and shade the solution region.

Do — Determine if a given point is in the solution region of a system of inequalities or not.

Know — The solution of a system of linear inequalities is made up of all the points in the common solution region. The inequalities can be called constraints, and the region may be bounded or unbounded.

Date: _____

Lesson 6: Optimization Using Systems of Inequalities

Standard(s)

- **2A.3E** Formulate systems of at least two linear inequalities in two variables.
- **2A.3F** Solve systems of two or more linear inequalities in two variables.
- **2A.3G** Determine possible solutions in the solution set of systems of two or more linear inequalities in two variables.

Important Vocabulary

- system of linear inequalities
- solution set/region
- boundary line
- constraint
- **solution/shaded region**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- Each Student needs 3 colored pens (not red) or highlighters

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Debrief (5 min)
- Student Practice (20 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to find the maximum or minimum of an objective function given linear constraints.

Opportunities to CFU

- ✓ INM problems #4-7
- ✓ Debrief
- ✓ Student Practice #1

Other Notes to Inform Your Planning

Today we are combining all that we have learned this unit so far and applying it to optimization. Students have all the tools and information they need to complete this activity. You'll want to check in with groups of course, but we're really trying to let students work through this lesson on their own as much as possible. As groups complete questions, ask them to add their work to the board.

Lesson Look Fors

Look for teachers to…

- ❑ Let students do the work but highlight students' solutions.
- ❑ Support students in identifying the objective function and evaluating different points to find the maximum and minimum.

Look for students to…

- ❑ Generate and graph a system of inequalities and correctly identify the shaded region.
- ❑ Solve for the points of intersection of the constraints that bound the shaded region.

Student Know/Do Chart

Students can

Do — Graph a system of linear inequalities and shade the solution region.

Do — Find the maximum and minimum value if they exist for a given function $f(x, y)$.

Know — Know when optimizing an objective function that if there is a maximum or minimum to an objective function $f(x, y)$, then the max or min will come from the vertices of the shaded region.

Know — If the shaded region is unbounded then there will only be a minimum or a maximum, but not both when solving a given objective function $f(x, y)$.

Date: _____

Lesson 7: Introduction to Matrices

Standard(s)

2A.3B Solve systems of three linear equations in three variables by using technology with matrices ~~and substitution.~~

Important Vocabulary

- **matrix**
- **element**
- **dimension, m x n**
- **square matrix**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- TI-Nspire Calculators (1 per) or Desmos-Matrix Calculator

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Debrief (5 min)
- Student Practice (20 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to identify matrix elements and perform matrix addition, subtraction, and multiplication. In this lesson students will understand that a system of linear equations can be represented using a matrix.

Opportunities to CFU

- ✓ End of the INM
- ✓ Debrief
- ✓ Student Practice #1-2, 13-14

Focus on Disciplinary Literacy

Student Practice #13-15

Other Notes to Inform Your Planning

During the lesson students will use the calculator or Desmos Matrix Calculator on the computer to define matrices and perform operations such as addition, subtraction, and multiplications. This lesson introduces students to matrices which they will soon use to solve systems of linear equations. Students will only perform basic matrix operations by hand and then will use technology for solving systems with matrices. This topic is continued over the next two lessons.

Lesson Look Fors

Look for teachers to…

- ❑ Establish the definitions of matrices and support students in learning the algorithms of matrix operations.
- ❑ Support students with setting up matrices in their calculator.

Look for students to…

- ❑ Set up a matrix and identify a specific element.
- ❑ Add, subtract, and multiply matrices using technology.

Student Know/Do Chart

Students can

- Do: Identify a specific element of a given matrix.
- Do: Add or subtract given matrices when the dimensions match.
- Do: Determine if matrix multiplication can be performed by comparing the rows and columns of the matrices given, then complete the multiplication using technology.
- Know: Order matters in matrix multiplication. The columns of the first matrix must match the rows of the second matrix to multiply two matrices.

Date: _____

Lesson 8: Solving Linear Systems with Matrix Inverses & RREF

Standard(s)

2A.3B Solve systems of three linear equations in three variables by using technology with matrices ~~and substitution.~~

Important Vocabulary

- matrix
- element
- dimension, m x n
- square matrix
- **coefficient matrix**
- **variable matrix**
- **constant matrix**
- **extended matrix**
- **identity matrix**
- **inverse matrix**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- TI-Nspire Calculators (1 per) or Desmos-Matrix Calculator

Lesson Structure:

- Do Now (5 min)
- INM (20 min)
- Debrief (5 min)
- Student Practice (25 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson students will solve systems of linear equations using matrix inverses and RREF using the technology tools. Students will understand the different matrices that are created from the system of equations and will be able to solve the problems by using technology to find the inverse of the coefficients matrix and also using Reduced Row-Echelon Form.

Opportunities to CFU

✓ After INM #7 & 12

✓ Student Practice #1-3

Other Notes to Inform Your Planning

The lesson begins with the explanation of how to write a system of linear equations in two and three variables in matrix form. This is an exploration lesson. Allow time for students to get familiar with the different technology tools to be used. Also, ask questions to the students regarding the different matrices in all the problems. Here are some examples of the questions:

- What element is position A_{11}?
- What is the solution of the system of equations?
- What is the extended matrix for this problem?

Lesson Look Fors

Look for teachers to…

- ❑ Guide students in converting systems of linear equations into the appropriate matrices so students can solve them using inverse matrices and RREF functions on the calculator.
- ❑ Support students with using technology to solve systems with matrices.

Look for students to…

- ❑ Rewrite systems of linear equations so they can be set up as coefficient and constant matrices.
- ❑ Set up matrix equations using technology and the inverse of the coefficient matrix to find the solution of the linear system.

Student Know/Do Chart

Students can

Do: Solve a system of linear equations using matrices and technology.

Know: The solution to a matrix equation can be rewritten as an ordered pair or ordered triple as the solution of the original system of linear equations.

Know: If $AX = B$, then $X = A^{-1}B$ where A is the coefficient matrix, X is the variable matrix, B is the constant matrix, and A^{-1} is the inverse of matrix A.

Date: _____

Lesson 9: Writing and Solving Linear Systems with Matrix Inverses

Standard(s)

- **2A.3A** Formulate systems of equations, including systems consisting of three linear equations in three variables and systems consisting of two equations~~, the first linear and the second quadratic.~~
- **2A.3B** Solve systems of three linear equations in three variables by using technology with matrices and substitution.

Important Vocabulary

- matrix
- element
- dimension, m x n
- square matrix
- coefficient matrix
- variable matrix
- constant matrix
- extended matrix
- identity matrix
- inverse matrix

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- TI-Nspire Calculators (1 per) or Desmos-Matrix Calculator

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Debrief (5 min)
- Student Practice (20 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to write and solve systems of linear equations using matrix inverses to interpret real-world scenarios. After the students set up a system of equations, they will use matrices and technology to solve the systems and interpret their solutions in the context of the problems.

Opportunities to CFU

✓ Any INM problems

✓ Student Practice #1-2

Focus on Disciplinary Literacy

INM #1c & Debrief

Other Notes to Inform Your Planning

This lesson has students combine writing systems of linear equations to solve contextual problems and using matrices to solve systems. Students may use matrix inverses or the RREF function to solve the linear systems. Students will not be expected to use matrices by hand, only technology. This lesson is designed to help students see that matrices, especially when supported with technology, can be an effective way to solve problems involving linear systems.

Lesson Look Fors

Look for teachers to…

❑ Support students in connecting their work from lesson 4 to using matrices to solve the systems they write.

Look for students to…

❑ Write their own system of linear equations and utilize matrices and technology to solve the system and apply the solution to the context of the problem.

Student Know/Do Chart

Students can

Do — Write and solve three variable linear systems using matrices and the aid of technology.

Know — Know that to solve a system using matrices, you need to create the inverse of the coefficient matrix and multiply it by the constant matrix or create the augmented matrix and use the RREF function.

Know — Know that the answer to a system solved with matrices still needs to be written as an ordered pair, an ordered triple, or given in context of the scenario.

Recommended Unit 3 Success Days Material and Resources

Date: _____

To review topics taught up to 3.3, use the following resources. Your exit ticket data should be used to determine individualized needs. The resources can be used in small groups, whole groups, or independent groups and be integrated with other classroom routines, like computer aligned practice and teacher-led groups.

To review or practice solving linear systems by graphing, use...

Sample Activities & Tasks:

- Solving Systems by Graphing Investigation
- Solving Systems of Equations by Graphing Exploration and Extra Practice

Content Video Lessons:

- Solving Linear Systems by Using Graphs

To review or practice solving systems of linear equations algebraically, use...

Desmos Interactive Applets:

- Racing Dots Activity
- Wafers and Crème Activity

Sample Activities & Tasks:

- Solving Systems of Equations by Substitution Exploration and Extra Practice

To review topics taught up to 3.6, use the following resources. Your exit ticket data should be used to determine individualized needs. The resources can be used in small groups, whole groups, or independent groups and be integrated with other classroom routines, like computer aligned practice and teacher-led groups.

To review or practice writing & solving system of linear equations, use...

Sample Activities & Tasks:

- Tic Tac Toe Word Problem Activity
- The Storybook Diner Dilemma & Systems Word Problem Practice
- On Your Own: Formulating Systems of Linear Equations

Content Video Lessons:

- Systems of Equations with Elimination: Infinite Solutions

To review or practice solving systems of linear inequalities, use...

Sample Activities & Tasks:

- Systems of Linear Inequalities 1 TI-Calculator Activity
- Systems of Linear Inequalities 2 TI-Calculator Activity

Content Video Lessons:

- Graphing Systems of Linear Inequalities

To review or practice optimization using systems of linear inequalities, use...

Sample Activities & Tasks:

- Linear Programming Your Life (answers)
- Applications of Linear Systems TI-Calculator Activity
- Maximizing Your Efforts TI-Calculator Activity

Content Video Lessons:

- Graphing a Feasible Region
- Solving Linear Programming Problems
- Linear Programming

Date: _____

Unit 3 Exam

Standard(s)	Notes for Intellectual Preparation & Lesson Planning
◆ **2A.3A** Formulate systems of equations, including systems consisting of three linear equations in three variables and systems consisting of two equations~~, the first linear and the second quadratic.~~ **Problems #3, 7, 8** ◆ **2A.3B** Solve systems of three linear equations in three variables by using technology with matrices and substitution. **Problem #1, 2, 3, 5, 7, 8** ◆ **2A.3E** Formulate systems of at least two linear inequalities in two variables. **Problems #9a** ◆ **2A.3F** Solve systems of two or more linear inequalities in two variables. **Problems #6, 9b** ◆ **2A.3G** Determine possible solutions in the solution set of systems of two or more linear inequalities in two variables. **Problems #4, 9c**	**Necessary Materials and Pre-Lesson Prep** ▪ Algebra 2 Unit 3 Exam ▪ Assessment Companion for Algebra 2 Unit 3 Exam found on Curriculum Corner **Notes to Inform Your Planning** Review the Unit 3 Exam on Curriculum Corner. Internalize & create an exemplar for the assessment prior to teaching the unit as part of unpacking the unit. Use the exemplar to spar with the answer key provided on Curriculum Corner. Administer the Unit 3 Exam following guidance on the Scope & Sequence to ensure the scoring deadline is met.

UNPACKED STANDARDS

Focus standards for this unit.

Standards Clarification		
Standards	**Specificity**	**Notes/Explanations/Examples**

2A.3A Formulate systems of equations, including systems consisting of three linear equations in three variables and systems consisting of two equations~~, the first linear and the second quadratic.~~	Including, but not limited to:
	• Systems of linear equations
	  ○ Two equations in two variables
	  ○ Three equations in three variables
		• Grade Level(s):
		  ○ Algebra I solved systems of two linear equations in two variables using graphs, tables, and algebraic methods.
		  ○ Various mathematical process standards will be applied to this student expectation as appropriate.

2A.3B Solve systems of three linear equations in three variables by using technology with matrices and substitution.
(**NOTE:** AP readiness does not support inclusion of Gaussian elimination)
	Including, but not limited to:
	• 3 x 3 system of linear equations
	  ○ Three variables or unknowns
	  ○ Three equations
	• Standard form for systems of equations – variables on left side of the equal sign in alphabetical order with constant on the right side of the equal sign
	• Methods for solving systems of three linear equations in three variables
	  ○ Gaussian elimination – sequence of elementary row operations on a matrix of coefficients and answers to transform the matrix into row echelon form (ref)
	  ○ Technology with matrices
	  ○ Substitution
	  ○ Elimination
	Special cases
	  ○ All variables are eliminated
	  ▪ Infinite number of solutions – remaining constants yield a true statement
	  ▪ No solutions – remaining constants yield a false statement
		• Grade Level(s):
		  ○ Algebra I solved systems of two linear equations in two variables using graphs, tables, and algebraic methods.
		  ○ Various mathematical process standards will be applied to this student expectation as appropriate.

2A.3E Formulate systems of at least two linear inequalities in two variables.	Including, but not limited to:
	• Systems of linear inequalities in two variables
	  ○ Two variables or unknowns
	  ○ Two or more inequalities
	• Mathematical problem situations
	  ○ Graphical interpretation
	  ○ Verbal interpretation
	• Real-world problem situations represented by systems of inequalities
	  ○ Two linear inequalities
	  ○ Linear programming problem situations
		• Grade Level(s):
		  ○ Algebra I wrote linear inequalities in two variables given a table of values, a graph, and a verbal description.
		  ○ Algebra I solved systems of two linear inequalities in two variables using graphs, tables, and algebraic methods.
		  ○ Various mathematical process standards will be applied to this student expectation as appropriate.

Standards Clarification		
Standards	**Specificity**	**Notes/Explanations/Examples**

2A.3F Solve systems of two or more linear inequalities in two variables.	Including, but not limited to:
	• Systems of linear inequalities in two variables
	  ○ Two variables or unknowns
	  ○ Two or more inequalities
	• Method for solving system of inequalities
	  ○ Graphical analysis of system
	• Representation of the solution as points in the solution region
		• Grade Level(s):
		  ○ Algebra I solved systems of two linear inequalities in two variables using graphs, tables, and algebraic methods.
		  ○ Various mathematical process standards will be applied to this student expectation as appropriate.

2A.3G Determine possible solutions in the solution set of systems of two or more linear inequalities in two variables.	Including, but not limited to:
	• Method for solving system of inequalities
	  ○ Graphical analysis of system
	• Methods for solving linear programming problem situations
	  ○ Graphical analysis of system
	• Representation of the solution as points in the solution region
	• Justification of solutions to system of inequalities
	• Justification of reasonableness of solution in terms of real-world problem situations
		• Grade Level(s):
		  ○ Algebra I solved systems of two linear inequalities in two variables using graphs, tables, and algebraic methods.
		  ○ Various mathematical process standards will be applied to this student expectation as appropriate.

VERTICAL STANDARDS

This section details the **prior standards** that link to the standards of this unit. Systems of Equations are not directly taught in later high school courses.

Algebra 1	Algebra 2
• **A.2I** Write systems of two linear equations given a table of values, a graph, and a verbal description. • **A.3F** Graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist. • **A.3G** Estimate graphically the solutions to systems of two linear equations with two variables in real-world problems. • **A.3H** Graph the solution set of systems of two linear inequalities in two variables on the coordinate plane. • **A.5C** Solve systems of two linear equations with two variables for mathematical and real-world problems.	• **2A.3A** Formulate systems of equations, including systems consisting of three linear equations in three variables and systems consisting of two equations~~, the first linear and the second quadratic.~~ • **2A.3B** Solve systems of three linear equations in three variables by using technology with matrices and substitution. (**NOTE:** AP readiness does not support inclusion of Gaussian elimination) • **2A.3E** Formulate systems of at least two linear inequalities in two variables. • **2A.3F** Solve systems of two or more linear inequalities in two variables. • **2A.3G** Determine possible solutions in the solution set of systems of two or more linear inequalities in two variables.